King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer semester 2015-2016
Major Exam #1, Saturday July 30, 2016
Time: **100** Minutes

Name: ____

ID#: ______________________ **Section:** (8:10-9:10 9:20-10:20)

Instructions:
1. The exam consists of 6 pages, including this page, containing 7 questions.
2. Answer all 7 questions. Show all the steps.
3. Make sure your answers are **clear** and **readable**.
4. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
5. If there is no space on the front of the page, use the back of the page.

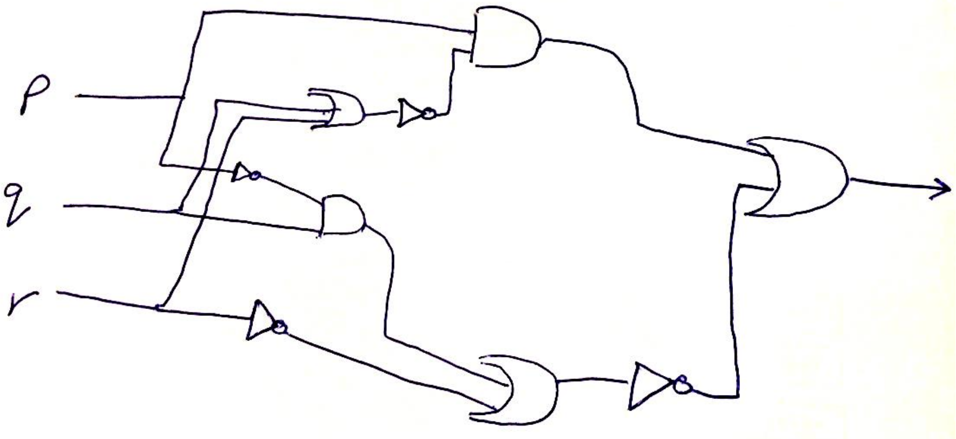
Question	Maximum Points	Earned Points	Remarks
1	20		
2	15		
3	10		
4	10		
5	20		
6	10		
7	15		
Total	**100**		

Rules of Inference:

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution
$\forall x P(x) \to P(a)$ for all a	Universal Instantiation	$\exists x P(x) \to P(a)$ for some a	Existential Instantiation
$P(a)$ for all a $\to \forall x P(x)$	Universal Generalization	$P(a)$ for some a $\to \exists x P(x)$	Existential Generalization

Q1: [20 points] Answer the following questions.

a) [6 points] Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg(q \vee r)) \vee \neg((q \wedge \neg p) \vee \neg r)$ from input bits p, q, and r.



b) [5 points] Show that ¬p → (q → r) and q → (p ∨ r) are logically equivalent without using truth tables.

$$
\begin{aligned}
q \rightarrow (p \vee r) &\Leftrightarrow \neg q \vee (p \vee r) \\
&\Leftrightarrow (\neg q \vee p) \vee r \\
&\Leftrightarrow (p \vee \neg q) \vee r \\
&\Leftrightarrow p \vee (\neg q \vee r) \\
&\Leftrightarrow p \vee (q \rightarrow r) \\
&\Leftrightarrow \neg p \rightarrow (q \rightarrow r)
\end{aligned}
$$

c) [6 points] Show that $((p \vee q) \wedge \neg p) \rightarrow q$ is a tautology without using a truth table.

$$
\begin{aligned}
&((p \vee q) \wedge \neg p) \rightarrow q \\
&\Leftrightarrow \neg((p \vee q) \wedge \neg p) \vee q \\
&\Leftrightarrow \neg((p \wedge \neg p) \vee (q \wedge \neg p)) \vee q \\
&\Leftrightarrow \neg(F \vee (q \wedge \neg p)) \vee q \\
&\Leftrightarrow \neg(q \wedge \neg p) \vee q \\
&\Leftrightarrow (\neg q \vee p) \vee q \\
&\Leftrightarrow (p \vee \neg q) \vee q \\
&\Leftrightarrow p \vee (\neg q \vee q) \\
&\Leftrightarrow p \vee T \Leftrightarrow T
\end{aligned}
$$

d) [3 points] State the inverse, contrapositive and converse of the following implication in the form "if …. then ….".
"You will never get elected unless a majority of people vote for you."

The above sentence is equivalent to:
If a majority of people does not vote for you, you will never get elected.

Inverse:
If a majority of people vote for you, you will get elected.

Contrapositive:
If you get elected, then a majority of people have voted for you.

Converse:
If you do not get elected then a majority of people have not voted for you.

Q2: [15 points] Consider the following statements: "If Salim takes the job offer then he will get a signing bonus." "If Salim takes the job offer, then he will receive a higher salary." "If Salim gets a signing bonus, then he will not receive a higher salary." "Salim takes the job offer."

a) [9 points] Translate the above statements to propositional logic.

Let p = Salim takes the job offer, q = Salim gets a signing bonus, and r = Salim receives a higher salary

1. $p \rightarrow q$
2. $p \rightarrow r$
3. $q \rightarrow \neg r$
4. p

b) [6 points] Determine whether these statements are consistent or not. Clearly justify your answer using logic.
From 4, p is true.
By 1, q must be true.
By 2, r must be true.
Now, Statement 3 is False. Therefore, the statements are not consistent.

Q3: [10 points] . On the island of knights and knaves, three inhabitants A, B and C are being interviewed. A and B make the following statements:

A: B is a knight.
B: If A is a knight so is C.

Can you determine what A, B and C are? Clearly justify your answer.

A: B is a knight.
B: A is a knight → C is a knight.

Assume first that A is a knight. Then, By A's statement, B is a knight. Hence, by B's Statement, C is a knight has to be true. So, all of them are knights.

Assume now that A is a knave. Then, By A's statement, B is also a knave.
Now, since B is a knave, then B's statement has to false. The implication in B's statement can only be false if "A is a knight" is True and "C is a knight" is False. Which means A is a knight and C is a knave. This contradicts our assumption that A is a knave *Contradiction*.

Therefore, All of them are knights.

Q4: [10 points] Prove that if m and n are integers and mn is even, then m is even or n is even.

Let us do it by contrapositive (indirect proof). That is, we show that
If m is odd and n is odd, then mn is odd.
m is odd implies $m=2k+1$.
n is odd implies $n = 2j+1$.
$mn = (2k+1)(2j+1) = 4kj+2k + 2j + 1 = 2(2kj + k + j) + 1$, which is obviously odd.

Q5: [20 points] Let $S(x)$ be the statement "x is a student in my class", $F(x)$ is the statement "x has a facebook account" and $P(x, y)$ be the statement "x has shared a post by y," where the domain for the variables x and y consists of all people. Use quantifiers and predicates to express each of these statements, where no negation is outside a quantifier or an expression involving logical connectives. You are **not** allowed to use "∃!".

a) [4 points] None of the students in my class have a facebook account.

$$\neg\exists x(S(x) \land F(x)) \Leftrightarrow \forall x(S(x) \to \neg F(x))$$

b) [4 points] Ali has shared at least one post by every student in the class.

$$\forall y(S(y) \to P(Ali, y))$$

c) [4 points] All students in my class who have a facebook account have shared the same post by Khalid.

Not Graded. The answer is not possible with the given predicates. Its points go to every student in class.

d) [4 points] There is exactly one student in my class who has shared a post by himself.

$$\exists x(S(x) \land P(x,x) \land \neg\exists y(S(y) \land P(y,y) \land (y \neq x)))$$

$$\Leftrightarrow \exists x(S(x) \land P(x,x) \land \forall y(\neg S(y) \lor \neg P(y,y) \lor (y = x)))$$

$$\Leftrightarrow \exists x\Big(S(x) \land P(x,x) \land \forall y\big((S(y) \land P(y,y)) \to (y = x)\big)\Big)$$

$$\Leftrightarrow \exists x\Big(S(x) \land \forall y\big((S(y) \land P(y,y)) \leftrightarrow (y = x)\big)\Big)$$

e) [4 points] Someone has not shared a post with any student in my class who has a facebook account.

$$\exists x\Big(\forall y\big((S(y) \land F(y)) \to \neg P(x,y)\big)\Big)$$

Q6: [10 points] Answer the following questions.

a) (2 points) Find the truth set of $R(x)$: $x < x^2$ where the domain is the set of integers.

$$Z - \{0,1\}$$

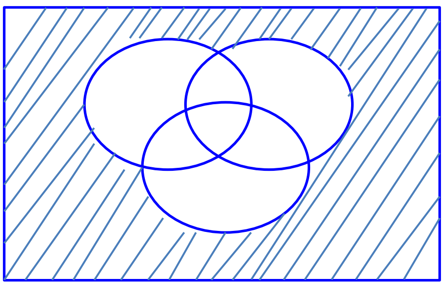
b) (2 points) Find the power set of $\{a, b, \{a, b\}\}$.

$$\{\Phi, \{a\}, \{b\}, \{\{a, b\}\}, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\}$$

c) (4 points) Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i, $A_i = [-i, i]$, that is the set of real numbers x with $-i \le x \le i$.

$\bigcup_{i=1}^{\infty} A_i = \mathcal{R}$ and $\bigcap_{i=1}^{\infty} A_i = [-1,1]$

d) (2 points) Draw the Venn Diagram for $\bar{A} \cap \bar{B} \cap \bar{C}$



Q7: [15 points] Use the rules of inference to show that:

(1) $\forall x\Big(\big(P(x) \vee Q(x)\big) \to R(x)\Big)$,
(2) $\forall x\big(R(x) \to S(x)\big)$,
(3) $\exists x\big(\neg S(x) \wedge T(x)\big)$

imply $\exists x \neg P(x)$.

From (3) using existential instantiation, there is an element in the domain c such that

$$\neg S(c) \wedge T(c) \ldots \ldots \ldots \ldots (4)$$

Using Simplification of (4)

$$\neg S(c) \ldots \ldots \ldots \ldots (5)$$

holds.

Using Universal Instantiation and modus tollens of (2) with (5) we get

$$\neg S(c) \to \neg R(x) \ldots \ldots \ldots (6)$$

Using Universal Instantiation and modus tollens of (1) with (6) we get

$$\neg R(c) \to \neg\big(P(c) \vee Q(c)\big)$$
$$\Leftrightarrow \neg R(c) \to \big(\neg P(c) \wedge \neg Q(c)\big) \ldots \ldots \ldots \ldots (7)$$

Using simplification of (7)

$$\neg P(c) \ldots \ldots \ldots \ldots (8)$$

holds.
Using existential generalization of (8) we get $\exists x \neg P(x)$.